CS 6840 – Algorithmic Game Theory (3 pages) Spring 2012

Lecture 32 Scribe Notes

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1 Today

- Next few lectures will cover General Second Price Auctions (GSP)
- Ideas for paid advertising
- How to generalize the idea into a GSP auction
- Based on paper by Edelman, Ostrowski, and Schwartz

1.1 History of GSP Auctions

- Goto was the first company to allow sponsored listings for search terms
- The higher you pay, the higher you appear on the search
- Now search is divided into sponsored and free listings

1.2 Original Version

- First price - you pay what you bid
  - First price makes it hard to bid and creates problems with equilibria
- pay for clicking on ad, not just showing the ad (pay-per-click)
  - pay-per-impression - everytime the ad is shown you pay
  - pay-per-action - when someone buys on a redirect from the search you pay
- Sort by price offered

1.3 Move to GSP

- Given the problems with the original version GSP was the accepted way within a few years
- Still offer a bid - willingness to pay for a click
- Sort by bids
- Everyone pays the amount of the next highest bid
- It looks like GSP will last as the dominant way to pay for search

1.3.1 Added feature of GSP

- GSP was misused by people bidding for irrelevant search terms (Example: 'Stella's Coffee' bidding for term 'hotels in ithaca'). These people would pay a little amount as those searching are unlikely to click, however the slot still has value to it.
- Google added a quality factor which is a parameter of the advertisement and the search term
  - Quality factor is denoted as γ - the probability that the ad gets clicked on if shown in the top slot
  - This factor is slot specific and we assume that the top slot is the most likely to be clicked on

2 GSP with Quality factors

2.1 Setup

- Ask users for bids b_i
- Compute quality factors γ_i
- Sort by $\gamma_i \cdot b_i$
- per click price = critical value
  - We have $\gamma_i \cdot b_i$ in slot i and $\gamma_{i+1} \cdot b_{i+1}$ in slot $i+1$
  - $\gamma_i \cdot b_i \geq \gamma_{i+1} \cdot b_{i+1}$ so critical value, or smallest price to keep the slot is $p_i = \frac{\gamma_{i+1} \cdot b_{i+1}}{\gamma_i}$

For simplicity we assume that $\gamma_i = 1$ for all i. This assumption is used in most research. When there is no loss of generality the results extend to the case with different quality factors, however, the formulas get much more complicated to deal with.

2.2 Relating to our Auction Framework

- Must add a discount factor to relate the benefit of being in the top slot
- Model of clicks: $1 \geq \alpha_1 \geq ... \geq \alpha_n$, α_i is associated with slot i
- Probability that ad i on slot j gets clicked is $\alpha_j \gamma_i$ (These are seperable)
- Lets us assume that the lower you are the worse off you are

2.3 A Game Framework

- Stratey is your bid, γ and α are known
- Value v_i = value for click for advertiser
- Side Note: Isn't value dependent on the slot?
  - Answer: Yes, but for simplicity assume that v_i is the same for all slots j

2.3.1 Consider the special case with only 1 slot

- Remaining slots have $\alpha = 0$
- This results in a regular second price auction with truthful bidding
  - Bidding $b_i = v_i$ for a click is the dominant strategy

2.3.2 Case with 2 slots

Is it a good strategy to bid above your value?

- $b_i > v_i$ is dominated by $b_i = v_i$, so no it is not
- We therefore assume that no players bid above their value (players are risk adverse)

Is it a good strategy to bid below your value?

- Yes, it could be a good idea

- Example: $\alpha_1 = 1$, $\alpha_2 = \frac{1}{2}$ and $v_1 = 1 - \epsilon$, $v_2 = \frac{1}{2}$
- Assuming player 2 bids truthfully ($b_2 = \frac{1}{2}$) it is better for player 1 to bid $b_1 = \frac{1}{2} - \epsilon$, gets half the clicks but gets them for free
- Therefore truthful bidding is not an equilibria

2.4 Questions of Equilibria

What is the equilbrium? Does an equilibria exist? How good is the equilibria?

- Equilbria does exist. The theorem comes directly from the paper that there exists an equilibria which is socially optimal so the price of stability is 1. We use price equilibrium in our proof

2.4.1 Price equilbria

- Post prices for slots $p_1, ..., p_n$ stating the price to pay to get the slot
- Equilibria exists if there exists a matching of ads to slots such that b_i is matched to favorite slot
- All players must choose different slots but can have multiple favorites (Will be discussed further in later lectures)
- Price equilibria must exist

2.4.2 Relating to GSP

- Bidders sorted in order $b_1, ..., b_n$
- b_i assigned to slot i
- b_i has value v_i
- $\forall i \forall j : (v_i - p_i)\alpha_i \geq (v_i - p_j)\alpha_j$
- We claim that the higher slot is more expensive
  - In every price equilibria it must be the case that $\forall i : p_i \geq p_{i+1}$
  - Only way to allow tradeoff for having a lower slot

In order to create a GSP where outcome is a price equilbria make $b_1 = v_1$ and $b_i = p_{i-1}$ for all other i. The outcome then becomes the posted prices.

2.4.3 Closing Fact

All price equilibrias are socially optimal which we will talk about in the following lectures